Energy-Efficient Data Forwarding for State Estimation in Multi-Hop Wireless Sensor Networks

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***Abstract*—State estimation is of great importance in various applications based on wireless networked control systems. Wireless sensor node usually forwards its data to a remote receiver through a series of relay nodes in order to save its limited energy. In this technical note, we investigate the data forwarding strategy design for accurate remote state estimation in multi-hop wireless networks. Based on different computational capability of relay nodes, we first propose two relay strategies, namely, Direct Forwarding Strategy (DFS) and Local Processing and Forwarding Strategy (LFS). Stability condition, estimation performance and energy efficiency of both the two strategies are theoretically analyzed. We further propose an Event-triggered Forwarding Strategy (EFS) which is able to balance the estimation accuracy and relay energy consumption. Numerical examples are employed to demonstrate the effectiveness of our design.**

***Index Terms*—Data forwarding, estimation, Kalman filtering, sensor networks.**

I. Introduction

Wireless networked control systems have attracted much interest in the past decade [2], [3]. With rapid development of electronic technology, most modern wireless sensor nodes have the ability to process certain computational tasks in addition to perceiving the area of interest and communicating wirelessly [4]. However, random packet drop is one of the main problems in wireless networked control systems, which may cause performance degradation or even system instability.

State estimation over wireless sensor networks with packet drops has been extensively studied in recent years. However, most of the previous works focused on single-hop communication [5]–[8]. In the real world, estimation via multi-hop is more practical especially for large scaled networks, and remains a challenging problem since both the system stability and estimation accuracy rely on the communication quality of each relevant link in the network.

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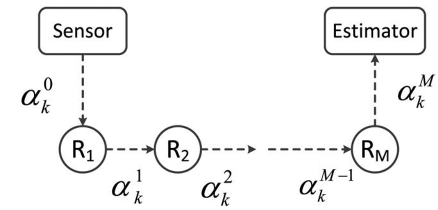

Fig. 1. System block diagram.

In this technical note, we consider the state estimation over multi-hop wireless networks subject to random packet drops. Specifically, we investigate how the data forwarding strategy design affects the performance of remote state estimation. Based on whether the relay nodes have sufficient computational capability, two typical relay strategies are proposed, namely Direct Forwarding Strategy (DFS) and Local Processing and Forwarding Strategy (LFS). Motivated by their respective advantages, we further propose a novel Event-Triggered Forwarding Strategy (EFS), which is able to achieve a good balance between the estimation accuracy and the energy consumption of relay nodes.

The rest of this technical note is organized as follows. We introduce our system framework and formulate the problem in Section II. Two typical relay strategies and the analytical comparison between them are described in Section III. We propose our novel event-triggered data forwarding strategy and analyze its performance in Section IV, and then evaluate its performance by numerical examples in Section V. Finally, the technical note is concluded in Section VI.

II. Problem Statement

Consider a discrete linear time-invariant system with the following dynamics:

$$x_k = Ax_{k-1} + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ and $y_k \in \mathbb{R}^m$ respectively represent the system state and sensor's measurement. $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are zero-mean Gaussian noises with covariances of $\mathbb{E}[w_k w_k^T] = Q \geq 0$ and $\mathbb{E}[v_k v_k^T] = R > 0$, respectively. We assume that w_k and v_k are mutually independent. And we further assume that A is unstable, $(A, \sqrt{Q})$ are controllable and (C, A) are observable.

We assume that the sensor node has the ability to implement a local Kalman filter, whose detailed process can be found in [9]. Then, the estimate $\hat{x}_k$ is transmitted to the remote estimator for further processing via wireless medium with successive M relay nodes, see Fig. 1. Since long-term monitoring is considered in this technical note, we assume that the estimation of all the sensors have reached the steady state, and further denote the converged estimation error covariance by $\bar{P}$, which has been proved to be the unique positive definite solution of the following algebraic Riccati equation [10]:

$$X = h(X) - h(X)C^T \left(Ch(X)C^T + R\right)^{-1} Ch(X) \tag{3}$$

where $h(X) \triangleq AXA^T + Q$. We define $h^2(X) \triangleq h \circ h(X) \triangleq h(h(X))$, and hence $h^t(X) \triangleq \underbrace{h \circ h \circ \cdots \circ h}_{t \text{ times}}(X)$. From [7], we have following lemma.

Lemma 2.1: If A is unstable and $Q \geq 0$, then the following statements hold.

1) $\bar{P} < h(\bar{P})$.
2) $h^t(X) < h^t(Y), t \in \mathbb{N}$, if $X < Y$.

We denote the arrival process of sensor's packet at time k by a binary value α_k^0, where $\alpha_k^0 = 1$ represents that R_1 successfully receives the sensor's packet at time k and $\alpha_k^0 = 0$ otherwise. Similarly, α_k^i characterizes the arrival process of the packet transmitted by the relay node R_i at time k. We assume that $\{\alpha_k^i\}$ are i.i.d., and denote the successful packet arrival probability by $\alpha^i \triangleq \Pr\{\alpha_k^i = 1\}$. TCP-like protocol [11] is considered throughout the technical note, i.e., successful arrival of packet is acknowledged without error at the receiver. We further assume that in-network processing is much faster than the dynamics of the system, i.e., the time delay that occurs during the multi-hop transmission between the sensor and the remote estimator is neglected [12].

In the remainder of this technical note, we assume all the relay nodes have the initial estimation as the same as that in the remote sensor, and thus have the initial estimation error covariance $\overline{P}$. Then, we first propose two typical strategies for the relay nodes with/without computational capabilities, then try to design a novel strategy to forward the sensors data in an energy-efficient way.

III. Direct Forwarding Strategy Versus Local Processing and Forwarding Strategy

A. Direct Forwarding Strategy (DFS)

DFS is designed for the relay nodes without sufficient computational capability, where each of them just simply forwards the received packets. It means that if $\alpha_k^{i-1} = 1$, then R_i will forward the received packet to R_{i+1}, otherwise it will remain idle for saving energy. In other words, the remote estimator will receive the current state estimate from the sensor at time k if and only if $\alpha_k^{s1} \triangleq \prod_{i=0}^{M} \alpha_k^i = 1$,[1] otherwise it will just make a time update of standard Kalman filter based on its local estimate according to $\bar{x}_k = A\bar{x}_{k-1}$, and accordingly, the final error covariance of DFS, which we denote by P_k^{s1}, is updated as

$$P_k^{s1} = \alpha_k^{s1}\bar{P} + \left(1 - \alpha_k^{s1}\right) h\left(P_{k-1}^{s1}\right). \tag{4}$$

Then we have $\mathbb{E}[P_k^{s1}] = P(\alpha_k^{s1} = 1)\overline{P} + P(\alpha_k^{s1} = 0)h(\mathbb{E}[P_{k-1}^{s1}])$. Thus, $\mathbb{E}[P_{k-1}^{s1}]$ is bounded with the probability 1 if and only if $\mathbb{E}[P_k^{s1}]$ is bounded. In the rest of this technical note, we will say the estimation with the error covariance is P_k^{e}[2] on the remote estimator is mean square stable if and only if $\mathbb{E}[P_k^e] < \Gamma$ for some finite $\Gamma > 0$ and any $k > 0$.

Theorem 3.1: Consider the system (1), (2) and all the relay nodes run DFS. The estimation on remote estimator is mean square stable if and only if

$$\alpha^{s1} > 1 - \frac{1}{\max_j |\lambda_j(A)|^2} \tag{5}$$

where $\alpha^{s1} \triangleq \prod_{i=0}^{M} \alpha^i$ and $\lambda_j(A)$ is all the eigenvalues of A.

[1]In order to distinguish between the transmission strategies in this note, we here use the superscript $s1$ for DFS. The other proposed strategies will be treated similarly.

[2]P_k^e is a general symbol of the estimation error covariance for any transmission strategy, while P_k^{s1}, P_k^{s2}, and P_k^{s3} are typically for DFS, LFS, and EFS throughout our manuscript.

Proof: Note that the final estimation error covariance is updated according to (4), then by taking expectations on both sides of which, we have

$$\begin{aligned}\mathbb{E}\left[P_k^{s1}\right] &= \alpha^{s1}\bar{P} + (1 - \alpha^{s1})h\left(\mathbb{E}\left[P_{k-1}^{s1}\right]\right) \\ &= \left(\sqrt{1 - \alpha^{s1}}A\right)\mathbb{E}\left[P_{k-1}^{s1}\right]\left(\sqrt{1 - \alpha^{s1}}A\right)^T \\ &\quad + \left[\alpha^{s1}\bar{P} + (1 - \alpha^{s1})Q\right].\end{aligned}$$

Define $\tilde{h}(X) = (\sqrt{1 - \alpha^{s1}}A)X(\sqrt{1 - \alpha^{s1}}A)^T + [\alpha^{s1}\bar{P} + (1 - \alpha^{s1})Q]$, where $X \geq 0$. Since $\alpha^{s1}\bar{P} + (1 - \alpha^{s1})Q > 0$, and A is unstable, $\tilde{h}(X)$ is a Lyapunov operator and monotonically increasing.

For the sequence $\{\mathbb{E}[P_k^{s1}]\}$, we have $\mathbb{E}[P_0^{s1}] = \bar{P}$, and

$$\begin{aligned}\mathbb{E}\left[P_1^{s1}\right] - \mathbb{E}\left[P_0^{s1}\right] &= \alpha^{s1}\bar{P} + (1 - \alpha^{s1})h(\bar{P}) - \bar{P} \\ &= (1 - \alpha^{s1})\left[h(\bar{P}) - \bar{P}\right] > 0.\end{aligned}$$

Therefore, from Lemma 2 in reference [5], $\{\mathbb{E}[P_k^{s1}]\}$ is monotonically increasing, i.e., $\mathbb{E}[P_{k+1}^{s1}] \geq \mathbb{E}[P_k^{s1}]$, $\forall k$. Thus $\{\mathbb{E}[P_k^{s1}]\}$ is mean square stable, namely $\mathbb{E}[P_k^{s1}] < \Gamma$, is equivalent to $\{\mathbb{E}[P_k^{s1}]\}$ converges, whose limit is defined as $\mathbb{E}[P^{s1}]$. Then by the continuity of $\tilde{h}$, we obtain $\mathbb{E}[P^{s1}] = \tilde{h}[\mathbb{E}[P^{s1}]]$, which means that $\mathbb{E}[P^{s1}]$ is the solution of the following Lyapunov matrix equation:

$$X - \left(\sqrt{1 - \alpha^{s1}}A\right)X\left(\sqrt{1 - \alpha^{s1}}A\right)^T = \alpha^{s1}\bar{P} + (1 - \alpha^{s1})Q. \tag{6}$$

It is well known that (6) has a unique strictly positive definite solution if and only if

$$\max_i \left|\lambda_i\left(\sqrt{1 - \alpha^{s1}}A\right)\right|^2 < 1. \tag{7}$$

Therefore, (5), equivalent to (7), is a necessary and sufficient condition of mean square stability on the remote estimator, because of the equivalence of each step above. ■

B. Local Processing and Forwarding Strategy (LFS)

Different from DFS, relay nodes will buffer the latest received data and have sufficient computational capability to implement a local estimation of the system state and buffer the result. More specifically, the estimate on the relay node R_i under this strategy is given by

$$\hat{x}_k^i = \alpha_k^{i-1}\hat{x}_k^{i-1} + \left(1 - \alpha_k^{i-1}\right)A\hat{x}_{k-1}^i \tag{8}$$

while the corresponding estimation error covariance is given by

$$P_k^i = \alpha_k^{i-1}P_k^{i-1} + \left(1 - \alpha_k^{i-1}\right)h(P_{k-1}^i) \tag{9}$$

where with a little abuse of notation, the remote estimator is denoted by $i = M + 1$, i.e., $P_k^{s2} \triangleq P_k^{M+1}$. Notice that (8) has been shown to be the optimal least-square estimation under packet-dropping communication [13]. In this case, every relay node tries its best to send the latest estimate to its son node.

Property 3.1: Under LFS, for any given two relay nodes R_i and R_j, where $i < j$, their estimation error covariances satisfy $P_k^i \leq P_k^j$.

Proof: It is sufficient to prove that $P_k^i \leq P_k^{i+1}$, $\forall i \geq 0$. To this end, induction method is used. First, it is obvious that $\bar{P} = P_k^0 \leq P_k^1$. Second, assume $P_k^{i-1} \leq P_k^i$ for a given i and any k. When $\alpha_k^i = 1$, $P_k^{i+1} = P_k^i$. Define $\tau_k^i = \max\{t | \alpha_t^i = 1, t\epsilon[1, k]\}$. When $\alpha_k^i = 0$, we have

$$P_k^{i+1} = h^{k-\tau_k^i}\left(P_{\tau_k^i}^i\right), \; P_k^i = h^{k-\tau_k^{i-1}}\left(P_{\tau_k^{i-1}}^{i-1}\right). \tag{10}$$

Then, from (10), if $\tau_k^i \geq \tau_k^{i-1}$, we have

$$P_k^{i+1} = h^{k-\tau_k^i}\left(h^{\tau_k^i - \tau_k^{i-1}}\left(P_{\tau_k^{i-1}}^i\right)\right) = h^{k-\tau_k^{i-1}}\left(P_{\tau_k^{i-1}}^i\right)$$

and else if $\tau_k^i < \tau_k^{i-1}$, we have

$$P_k^{i+1} = h^{k-\tau_k^{i-1}}\left(h^{\tau_k^{i-1}-\tau_k^i}\left(P^i_{\tau_k^i}\right)\right) \geq h^{k-\tau_k^{i-1}}\left(P^i_{\tau_k^{i-1}}\right).$$

Therefore, one can find that

$$P_k^{i+1} - P_k^i \geq h^{k-\tau_k^{i-1}}\left(P^i_{\tau_k^{i-1}} - P^{i-1}_{\tau_k^{i-1}}\right) \geq 0$$

which is due to the assumption above and Lemma 2.1. Finally, we can conclude according to the induction that $P_k^i \leq P_k^{i+1}$. ■

Property 3.2: Under LFS, the expected estimation error covariance on relay node R_i, i.e., $\mathbb{E}[P_k^i]$, is monotonically increasing. $\mathbb{E}[P_k^i]$ is mean square stable, if and only if $\mathbb{E}[P_k^i]$ converges.

Proof: By taking expectation over (9), we have

$$\mathbb{E}\left[P_k^i\right] = \alpha^{i-1}\mathbb{E}\left[P_k^{i-1}\right] + (1-\alpha^{i-1})h\left(\mathbb{E}\left[P_{k-1}^i\right]\right). \quad (11)$$

Thus, $\mathbb{E}[P_{k+1}^i] - \mathbb{E}[P_k^i] = \alpha^{i-1}(\mathbb{E}[P_{k+1}^{i-1}] - \mathbb{E}[P_k^{i-1}]) + (1-\alpha^{i-1})(h(\mathbb{E}[P_k^i]) - h(\mathbb{E}[P_{k-1}^i]))$. Similar to the proof of Theorem 3.1, we have $\mathbb{E}[P_{k+1}^1] - \mathbb{E}[P_k^1] \geq 0$. Furthermore, $\mathbb{E}[P_1^i] \geq \mathbb{E}[P_0^i]$, for $i \in \{0, 1, \ldots, m+1\}$. Then, by induction method, $\mathbb{E}[P_k^i]$ is monotonically increasing. Consequently, the mean square stability and the convergence of $\mathbb{E}[P_k^i]$ are equivalent. ■

The stability condition for LFS is given in the following.

Theorem 3.2: Consider the system (1), (2) and all the relay nodes run LFS. The estimation on the remote estimator is mean square stable if and only if

$$\alpha^i > 1 - \frac{1}{\max_j |\lambda_j(A)|^2}, \quad \forall i \in \{0, 1, \ldots, M\}. \quad (12)$$

Proof: From Property 3.2, for the limit of $\mathbb{E}[P_k^i], k = 1, 2, \ldots,$ we have $\mathbb{E}[P^i] = \alpha^{i-1}\mathbb{E}[P^{i-1}] + (1-\alpha^{i-1})h(\mathbb{E}[P^i])$. Similar to that in Theorem 3.1, and noting that $\mathbb{E}[P^i] > 0$ if it exists, this theorem can be derived by induction method and the detail of proof is omitted due to the space limitation. ■

C. Comparison of Two Strategies

The following theorem shows the different performance between DFS and LFS in terms of the expectation of estimation error covariance.

Theorem 3.3: Suppose that the packet reception rates of the transmission links satisfy (5), the expected estimation error covariances on the remote estimator for DFS and LFS satisfy $\mathbb{E}[P^{s1}] \geq \mathbb{E}[P^{s2}]$.

Proof: From (11), and using recursive method on $\mathbb{E}[P_k^m]$, we have

$$\begin{aligned}
\mathbb{E}[P_k^{s2}] &= \mathbb{E}\left[P_k^{m+1}\right] = \alpha^m\mathbb{E}\left[P_k^m\right] + (1-\alpha^m)h\left(\mathbb{E}\left[P_{k-1}^{s2}\right]\right)\\
&= \alpha^0\alpha^1\cdots\alpha^m\mathbb{E}\left[P_k^0\right] + \sum_{i=1}^{m}\prod_{j=1}^{i}\alpha^m(1-\alpha^{m-j})h\left(\mathbb{E}\left[P_{k-1}^{m-i+1}\right]\right)\\
&\quad + (1-\alpha^m)h\left(\mathbb{E}\left[P_{k-1}^{s2}\right]\right)\\
&\leq \alpha^{s1}\bar{P} + \sum_{i=1}^{m}\prod_{j=1}^{i}\alpha^m(1-\alpha^{m-j})h\left(\mathbb{E}\left[P_{k-1}^{s2}\right]\right)\\
&\quad + (1-\alpha^m)h\left(\mathbb{E}\left[P_{k-1}^{s2}\right]\right)\\
&= \alpha^{s1}\bar{P} + (1-\alpha^{s1})h\left(\mathbb{E}\left[P_{k-1}^{s2}\right]\right).
\end{aligned}$$

The inequality above is obtained from the fact that $\mathbb{E}[P_{k-1}^i] \leq \mathbb{E}[P_{k-1}^{s2}]$, $i \geq 1$, which is straightforward from Property 3.1. Thus, there exists a sequence defined by $\{\mathbb{E}[\tilde{P}_k^{s2}]\}$ which satisfies $\mathbb{E}[\tilde{P}_0^{s2}] = \bar{P}$, $\mathbb{E}[\tilde{P}_k^{s2}] = \alpha^{s1}\bar{P} + (1-\alpha^{s1})h(\mathbb{E}[\tilde{P}_{k-1}^{s2}])$, and $\mathbb{E}[P_k^{s2}] \leq \mathbb{E}[\tilde{P}_k^{s2}]$, $k = 1, 2, \ldots$. Actually, we can find that $\{\mathbb{E}[\tilde{P}_k^{s2}]\}$ equals to $\{\mathbb{E}[P_k^{s1}]\}$. Therefore, $\mathbb{E}[P_k^{s2}] \leq \mathbb{E}[P_k^{s1}]$, and the proof is completed by taking the limit on this inequality. ■

Denote the energy consumption of transmitting a packet for a relay node by E, and the total energy consumption of DFS in time period k by J_k^{s1} and that of LFS by J_k^{s2}. Note that for DFS, a relay node R_j will launch a transmission only when $\prod_{i=0}^{j-1}\alpha_k^i = 1$. Therefore, the expected energy consumption of DFS in time period k is given by $\mathbb{E}[J_k^{s1}] = \sum_{j=1}^{M}(\prod_{i=0}^{j-1}\alpha^i)E$. On the other hand, for LFS, J_k^{s2} is constantly given by $J_k^{s2} = ME$. Obviously, DFS is likely to consume less energy than LFS. As a consequence, we conclude that LFS enhances the robustness of the system and improves the estimation performance at the expense of more energy consumption.

IV. Event-Triggered Forwarding Strategy

Note that in many cases, some transmissions in LFS are unnecessary. For instance, if at time k the estimate is successfully transmitted from the sensor to the remote actuator, i.e., all the relay nodes locally have the current system state estimate $\hat{x}_k$. Suppose that $\alpha_{k+1}^{i-1} = 0$, then according to LFS, the relay node R_i will update the local estimate to $A\hat{x}_k$ and transmit it to R_{i+1}. Then, if R_{i+1} receives the packet, it will update its estimate by $A\hat{x}_k$ and continue the transmission. However, if the packet from R_i is dropped, R_{i+1} will also make a local estimate as $A\hat{x}_k$. It is of interest that it makes no difference to the result whether the packet from R_i is successfully received or not. This motivates us to design a data forwarding strategy to launch the transmission only when it is necessary.

The overall structure of event-triggered forwarding strategy (EFS) is presented in Algorithm 1. We introduce an indicator $\lambda_k^i \in \{transmit, update\}$ to denote the decision made by R_i at time k. First of all, at any time slot, if one relay node receives a packet, it will replace the buffered estimate by the received one and forward it to its son node. It is because the latest received packet is more likely to contain more accurate estimate of the system state. Due to the error-free acknowledgements, R_i knows exactly the estimate and error covariance on its son node R_{i+1}. In case of no packet received, it will first implement a time update based on the buffered estimate, then compare the covariance gap with its son node R_{i+1} as

$$\Delta_k^i \triangleq P_{k-1}^{i+1} - P_{k-1}^i.$$

If it holds that[3] $Tr(\Delta_k^i) \geq \rho$ for a pre-given ρ, λ_k^i is set as *transmit*, otherwise $\lambda_k^i = update$.

Algorithm 1 Event-triggered Forwarding Strategy

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In every time period, each relay node does:
if R_i receives a packet then
    update the local estimate by x̂_k^i = x̂_k^{i-1}.
    set λ_k^i = transmit.
else
    update the local estimate by x̂_k^i = A x̂_{k-1}^i.
    calculate Δ_k^i = P_{k-1}^{i+1} − P_{k-1}^i.
    if Tr(Δ_k^i) ≥ ρ then
        set λ_k^i = transmit.
    else
        set λ_k^i = update.
    end if
end if
```

[3] $Tr(\Delta_k^i)$ should to be positive according to Property 3.1.

In the Algorithm 1, a relay node will not launch a transmission if it does not receive a packet at the current time instant and the benefit of a transmission is extremely low, where the benefit is characterized by the tunable value ρ. It is interesting to observe that if we set $\rho = 0$, the relay node will always launch a transmission and it becomes the LFS. On the other hand, $\rho = +\infty$ will results in no forwarding for a relay node unless it receives a packet from its parent node, i.e., it becomes DFS.

A. Stability Analysis

Since the proposed EFS will degrade into DFS when $\rho = 0$ and into LFS when $\rho = +\infty$, whose stability conditions are respectively given in Theorem 3.1 and Theorem 3.2, here we only consider the case when ρ is a finite positive value. Denote the error covariance on the remote estimator in EFS by P_k^{s3}, then the following theorem gives the condition for the mean square stability of EFS.

Theorem 4.1: Given that $0 < \rho < +\infty$, the estimation on the remote estimation is mean square stable if $\{\alpha^0, \alpha^1, \ldots, \alpha^M\}$ all satisfy (5).

Remark 4.1: Since it is challenging to obtain the necessary and sufficient condition of EFS stability, we only provide a sufficient condition of EFS stability in Theorem 4.1. In fact, this result is straightforward from Theorem 3.1 and Theorem 4.2.

B. Performance Analysis

In the proposed EFS strategy, the tunable value ρ plays an important role in our algorithm. Typically, ρ provides a tradeoff between the estimation performance and the transmission cost of the relay node. A bigger ρ allows larger tolerance of the estimate differences between neighboring relay nodes but substantially reduce in-network transmissions. On the other hand, a small ρ is beneficial to reducing the estimation error at the expense of more transmissions.

Property 4.1: Under EFS, for any given two relay nodes R_i and R_j, where $i < j$, their estimation error covariances satisfy

$$P_k^i \le P_k^j.$$

The proof of this property is the same with that of Property 3.1.

The following theorem compare the estimation performance among DFS, LFS and EFS.

Theorem 4.2: Given that $0 < \rho < +\infty$ and the final estimations of DFS, LFS and EFS are all mean square stable, the expected estimation error covariances for three strategies satisfy

$$\mathbb{E}[P^{s1}] \ge \mathbb{E}[P^{s3}] \ge \mathbb{E}[P^{s2}]$$

where the equalities hold only when $\alpha^i = 1,\ \forall i \in \{0, 1, \ldots, M\}$.

Proof: Redefine the decision indicator $\lambda_K^i \in \{transmit, update\}$ as $\lambda_k^i \in \{1, 0\}$, in which $\lambda_k^i = 1$ represents transmit; $\lambda_k^i = 0$ otherwise. Then under EFS, we have $P_k^0 = \overline{P}$ and

$$P_k^i = \lambda_k^{i-1}\alpha_k^{i-1}P_k^{i-1} + \left(1 - \lambda_k^{i-1}\alpha_k^{i-1}\right) h\left(P_{k-1}^i\right) \qquad (13)$$

where $i \in \{1, 2, \ldots, M+1\}$. Note that, for $k = 1, 2, \ldots, \lambda_k^0 = 1$ and thus $P_k^1 = \alpha_k^0 \overline{P} + (1 - \alpha_k^0)h(P_{k-1}^1)$.

First, we prove that $\mathbb{E}[P_k^{s3}] \ge \mathbb{E}[P_k^{s2}]$. In fact, from property 4.1, $P_k^{i-1} - h(P_{k-1}^i) \le h(P_{k-1}^{i-1}) - h(P_{k-1}^i) \le 0$ and $\lambda_k^i \le 1$. Thus for $i \ge 1$, from (13), we have

$$\begin{aligned} P_k^i &\ge h\left(P_{k-1}^i\right) + \alpha_k^{i-1}\left(P_k^{i-1} - h\left(P_{k-1}^i\right)\right) \\ &= \alpha_k^{i-1}P_k^{i-1} + \left(1 - \alpha_k^{i-1}\right) h\left(P_{k-1}^i\right). \end{aligned}$$

By taking expectation, we have

$$\mathbb{E}\left[P_k^i\right] \ge \alpha^{i-1}\mathbb{E}\left[P_k^{i-1}\right] + (1 - \alpha^{i-1})h\left(\mathbb{E}\left[P_{k-1}^i\right]\right). \qquad (14)$$

Therefore, we can conclude that there exists a sequence $\{\mathbb{E}[S_k^i], i \in \{0, 1, \ldots, M+1\}, k \ge 1\}$ defined by

$$\mathbb{E}\left[S_k^i\right] = \alpha^{i-1}\mathbb{E}\left[S_k^{i-1}\right] + (1 - \alpha^{i-1})h\left(\mathbb{E}\left[S_{k-1}^i\right]\right)$$

with initial value $\mathbb{E}(S_0^i) = \overline{P}$ which must satisfy $\mathbb{E}[P_k^i] \ge \mathbb{E}[S_k^i]$. Actually, from the definition of $\{\mathbb{E}[S_k^i]\}$, one can easily find that $\mathbb{E}[S_k^{M+1}] = \mathbb{E}[P_k^{S2}]$ which leads to the right inequality of the result.

Second, we prove that $\mathbb{E}[P_k^{s3}] \le \mathbb{E}[P_k^{s1}]$. Note that, according to the mechanism of EFS, it must be true that $\lambda_k^i = 1$ if $\lambda_k^{i-1} = 1$. Then from (13), we have

$$\begin{aligned} P_k^{s3} = P_k^{M+1} &= \lambda_k^M \alpha_k^M P_k^M + \left(1 - \lambda_k^M \alpha_k^M\right) h\left(P_{k-1}^{s3}\right) \\ &= \lambda_k^M \alpha_k^M \left(\lambda_k^{M-1}\alpha_k^{M-1}P_k^{M-1} + \left(1 - \lambda_k^{M-1}\alpha_k^{M-1}\right)\right. \\ &\qquad \left.\times h\left(P_{k-1}^M\right)\right) + \left(1 - \lambda_k^M \alpha_k^M\right) h\left(P_{k-1}^{s3}\right) \\ &= \lambda_k^{M-1}\alpha_k^{M-1}\alpha_k^M P_k^{M-1} + \left(\lambda_k^M \alpha_k^M - \lambda_k^{M-1}\alpha_k^{M-1}\alpha_k^M\right) \\ &\quad \times h\left(P_{k-1}^M\right) + \left(1 - \lambda_k^M \alpha_k^M\right) h\left(P_{k-1}^{s3}\right) \\ &\le \lambda_k^{M-1}\alpha_k^{M-1}\alpha_k^M P_k^{M-1} + \left(\lambda_k^M \alpha_k^M - \lambda_k^{M-1}\alpha_k^{M-1}\alpha_k^M\right) \\ &\quad + \left(1 - \lambda_k^M \alpha_k^M\right) h\left(P_{k-1}^{s3}\right) \\ &= \lambda_k^{M-1}\alpha_k^{M-1}\alpha_k^M P_k^{M-1} + \left(1 - \lambda_k^{M-1}\alpha_k^{M-1}\alpha_k^M\right) \\ &\quad \times h\left(P_{k-1}^{s3}\right) \le \cdots \le \lambda_k^0 \alpha_k^0 \alpha_k^1 \cdots \alpha_k^M P_k^0 \\ &\quad + \left(1 - \lambda_k^0 \alpha_k^0 \alpha_k^1 \cdots \alpha_k^M\right) h\left(P_{k-1}^{s3}\right) \\ &= \alpha_k^0 \alpha_k^1 \cdots \alpha_k^M \overline{P} + \left(1 - \alpha_k^0 \alpha_k^1 \cdots \alpha_k^M\right) h\left(P_{k-1}^{s3}\right). \end{aligned} \qquad (15)$$

By taking expectation over (15), it follows that:

$$\mathbb{E}\left[P_k^{s3}\right] \le \alpha^{s1}\overline{P} + (1 - \alpha^{s1})h\left(\mathbb{E}\left[P_{k-1}^{s3}\right]\right). \qquad (16)$$

Similar with the proof of theorem 3.3, the left inequality of the result must be right. It is obvious that only when $\alpha_k^i = 1$ for all k and i, which equals to the successful reception of each node at each time, the equalities hold. ■

Remark 4.2: It can be easily inferred from the above that the number of transmissions of EFS falls between that of DFS and LFS. Thus, EFS provides a balance between the estimation performance and the energy consumption. More specifically, if we simply use an arbitrary $\rho \in (0, Tr(h(\bar{P}) - \bar{P}))$, relay node will decide to remain idle once its local estimate is equal to that of its father node. It means that certain energy can be saved by eliminating the less useful transmissions without much performance degradation.By considering that once a packet is received, the corresponding covariance gap will be reset to 0 immediately, a relatively small ρ will be helpful to saving energy while protect the gap from being larger.

Remark 4.3: With Property 4.1 and the fact that the function $h^t(X)$ increases exponentially fast with t, we can infer that with a similar packet reception rate, the covariance gap bound of a latter relay node is easier to be triggered. This will make a latter relay node launch more transmissions and consume more energy by EFS, which is also verified in the simulation of Section V. To deal with this situation, we may increase the packet reception rate between the latter relay nodes or using a relatively larger ρ on such relay nodes.

V. Numerical Examples

Consider an example discrete-time system (1), (2) with the dynamics parameters given by

$$A = \begin{bmatrix} 1.3 & 0 \\ 0.2 & 1.1 \end{bmatrix}, \quad C = [1 \quad 1]$$

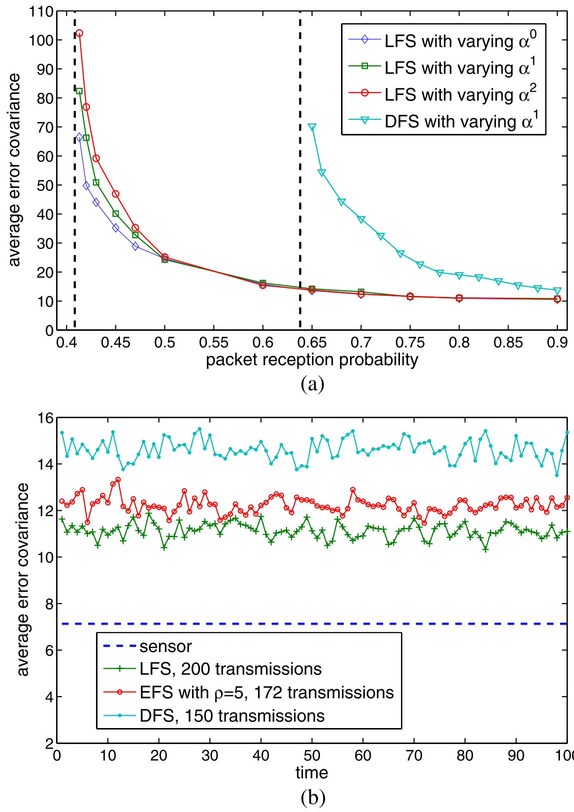

Fig. 2. Stability condition and estimation performance evaluation. (a) Stability analysis. (b) Estimation performance.

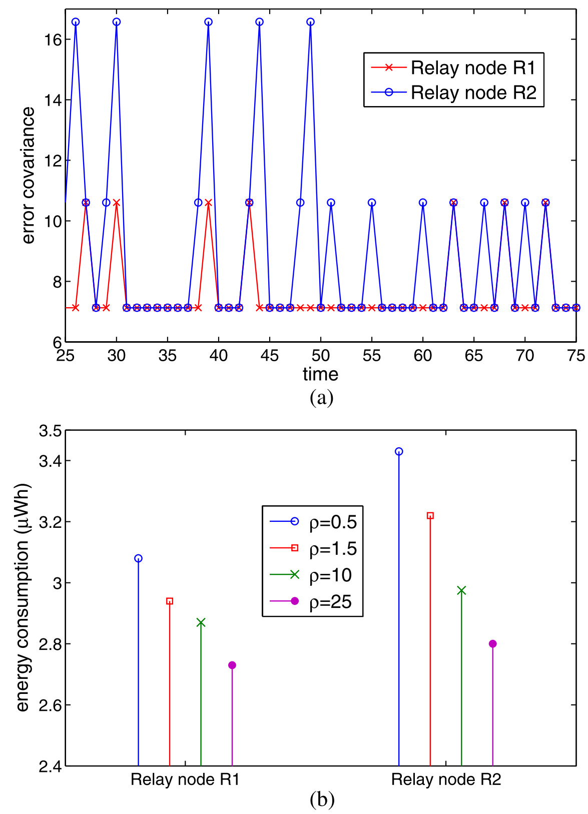

Fig. 3. Behaviors of relay nodes. (a) Error covariance. (b) Energy consumption.

with process noise covariance $Q = 0.5I$ and measurement noise $R = 2$. Two relay nodes R_1 and R_2 are employed to construct the multi-hop transmission from the sensor to the remote estimator. We use Telos node [14] throughout the evaluation, whose transmission power is 63 mW. The time for sending out one packet is set as 2 ms.

We first investigate the estimation stability under DSF and LFS in Fig. 2(a). We change one of packet reception rates of the three links in turn and set the other two as 0.8. Only one case for DSF is depicted because the other two curves exhibit very similar traces. First of all, we can find that the average error covariance of LFS is smaller than that of DFS for all feasible packet reception rates. With the decrease of packet reception rate, the average estimation error covariances of the two strategies will both increase. Especially, for LSF, when the packet reception rate falls to about $1 - (1/\max_i |\lambda_i(A)|^2) = 1 - 1/1.3^2 \approx 0.41$, the average estimation error covariance begins to diverge; while for DSF, the average covariance may become divergent when the packet reception rate approaches 0.64 which is equivalent to $\alpha^0\alpha^1\alpha^2 \to 1 - (1/\max_i |\lambda_i(A)|^2)$. We further compare the estimation performances of DFS, LFS and EFS in Fig. 2(b) with average results of 100 repeats. The packet reception rates are set as $\alpha^0 = 0.7$, $\alpha^1 = 0.8$, $\alpha^2 = 0.9$. It is easy to verify that in this case, the estimation is stable for all the three strategies, which is also consistent with the obtained stability conditions. One obvious result is that LFS achieves the best estimation performance with $\mathbb{E}[P^{s2}] \approx 11$, while for EFS with $\mathbb{E}[P^{s3}] \approx 12$ which is very close to LFS, and $\mathbb{E}[P^{s1}] \approx 15$ for DFS, which supports the result provided by Theorem 4.2. On the other hand, the average relay transmission numbers of LFS, EFS and DFS are 200, 172 and 150, which means that with an appropriate ρ, EFS achieves a satisfactory estimation performance while saving substantial energy.

Some behaviors of relay nodes R_1 and R_2 under our proposed EFS are shown in Fig. 3. We set $\rho = 10$ to see the evolution of error covariances of both the two relay nodes. It is found that the covariance of R_1 is always smaller than that of R_2 at any particular time, which verifies Property 4.1. We also find that the covariance of R_2 is well protected from getting larger due to such a ρ. Fig. 3(b) shows the average energy consumption of relay nodes R_1 and R_2 under different ρ with EFS strategy. It is found that quite different from DFS, the later relay R_2 with EFS will consume more energy than R_1. It is because the transmission decision of EFS is determined by corresponding covariance gap, while the gap of the later relay nodes will increase faster than the former one due to the property of function $h^t(\bar{P})$. We can also see that with larger ρ, the energy consumption of the relay will reduce. Furthermore, the reduction of R_2 is more remarkable than that of R_1 since the decision made by R_2 is more sensitive to ρ.

VI. Conclusion

In this technical note, we considered the design of sensor data forwarding strategies for state estimation in multi-hop wireless networks. We first propose and analyze two typical strategies, i.e., DFS and LFS, for two kinds of relay nodes, i.e., nodes with insufficient computation capability, and nodes with sufficient computation capability, respectively. We provide the analytical stability conditions for both forwarding strategies, and show that LFS always achieves better estimation accuracy than DFS but with more energy costs. Furthermore, we propose a novel EFS, which exploits the estimate difference between each pair of neighboring relay nodes to reduce the forwarding times while guaranteeing satisfactory estimation accuracy. The effectiveness of EFS is verified by numerical evaluation. It would be very interesting to further incorporate the coding strategies [15] or more complicated transmission patterns [16] to further improve the energy efficiency and system performance.

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