NACRP: A Connectivity Protocol for Star Topology Wireless Sensor Networks

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***Abstract*—Wireless sensor networks (WSNs) are an ever growing field of applications and one constituent of the future Internet-of-Things (IoT). In this work, we investigate star topology sensor networks compliant with the recent IEEE 802.15.4k standard in which sensors could fail to report sensing information to the access point (AP) due to temporary obstructions that clutter the link with the AP. The contribution of this work is twofold. First, we study general connectivity requirements in relay networks. Second, to restore connectivity and to recover from information loss, we propose the *neighbor-assisted connectivity recovery protocol* (NACRP), which automatically selects a subset of sensor nodes to act as relays for those which lack connectivity with the AP. In our study, we rely on the tool of stochastic geometry and in particular, on Poisson point processes to seek the tradeoff, which arises from the selection of a subset of relay nodes and the necessary transmitted power that relays need to use to restore network connectivity.**

***Index Terms*—Wireless sensor networks, star topology, mesh topology, MAC protocol, connectivity.**

I. Introduction

DURING past decades WSNs have witnessed a relentless research activity to leverage the deployment of low cost, easy to maintain and energy efficient solutions to monitor natural phenomena and men-made activities. Recently, the surge of packet data traffic over the cellular network has leveraged IoT and Machine-Type Communications, thus making sensors part of an omnipresent communication network [1]. Standardization bodies started several activities on WSN technology and the IEEE 802.15.4 and its subsequent amendments at both PHY and Medium Access Control (MAC) layers is one exemplary case of such ongoing effort [2].

We consider in this work the latest amendment IEEE 802.15.4k [3] for professionally installed star topology WSNs (STP-WSN). In STP-WSNs temporary obstructions might clutter the LOS connection between sensors deployed over a wide survey area and the central coordination point or AP. When this occurs, sensors will be unable to report sensed data although they function properly. Depending on the particular monitored phenomena, faulty sensors might trigger unnecessary human intervention or safety alarms.

Network connectivity is an important topic, studied in [4] by the critical transmission radius of a node, whilst [5] discusses the applicability of stochastic geometry. As mentioned, the family of IEEE 802.15.4 standards [6] has a crucial role in WSN technologies and specific studies for an IEEE 802.15.4k compliant STP-WSN are available already in [7], [8]. The NACRP developed for the first time in this work tackles the same general context of [8] but it provides a completely different solution since NACRP is a new protocol solving lack of connectivity under the centralized control of the AP.

A. Contribution of the Work

In this work we propose NACRP, a new protocol to automatically restore connectivity in an STP-WSN when obstructions clutter the communication link between sensors and the AP. The contribution of our work is twofold: i) we detail NACRP that, to the best of our knowledge, provides a clean slate solution to the problem of connectivity and ii) we investigate connectivity and the tradeoffs that arise from the adoption of the NACRP in the STP-WSN, relying on stochastic geometry and in particular on Poisson Point Processes (PPPs).

B. The IEEE 802.15.4k Standard

The IEEE 802.15.4k standard defines PHY and MAC layers specifications to support Low Energy Critical Infrastructure Monitoring (LECIM) networks. The standard supports simultaneous operation of at least 8 co-located orthogonal networks, with a transfer rate up to 40 kbits/s, minimum 1000 endpoints per AP and reliable operations in dramatically changing environments [3]. Channel time is organized in superframes, with each divided in several sub-beacon intervals (BIs) plus an optional inactive period delimited by the transmission of beacon frames transmitted by the AP. Beacons carry out general network information, as well as time synchronization for networked devices. The transmission of a beacon is followed by a Contention Access Period (CAP) and a Contention Free Period (CFP). During the CAP, carrier sense multiple access with collision avoidance (CSMA-CA) is used to transmit command frames for association and resource reservations inside the CFP. The CFP is TDMA based and is divided into guaranteed time slots (GTSs). During one GTS, only one sensor is allowed to communicate with the AP.

The remainder of this paper is organized as follows. Section II describes the features of the NACRP. Section III and Section IV provide the system model and analysis, respectively. Section V shows numerical results and in Section VI we draw the conclusions.

II. NACRP Protocol

We consider an STP-WSN in which the AP is also the network traffic sink, whilst sensors are deployed over a large survey area. The AP receives sensing data from the connected sensors unsolicited and depending on the specific monitored phenomena. Referring to Fig. 1, the position of the nodes is assumed known through a localization system (e.g. GPS service when available). The AP, upon missing sensing reports from one or more sensors for a period of time not less than a *Time-of-Failed-Report* (ToFR)[1], shall start the NACRP.

[1] The evaluation of the optimal ToFR duration is out of scope of this work.

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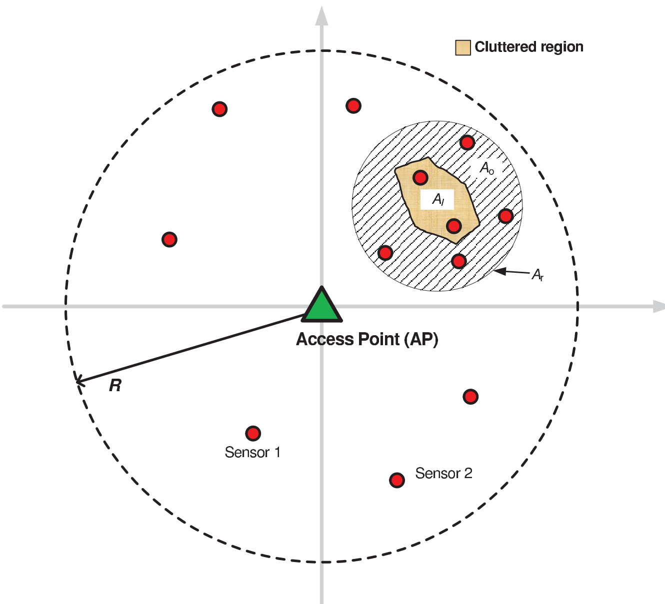


Fig. 1. Scenario where the cluttered region is modeled by a convex region.

Hence, the AP shall designate some of the sensors located in proximity of the cluttered region in Fig. 1 to start transmitting local beacons in order to create sub-networks. The AP shall inform the selected sensors whereby a new Information Element embedded in its beacon referred to as Sub-Net Information Element (SN-IE). The SN-IE will carry the identification of the selected sensor(s) and the time offset (with respect to the beginning of the superframe) required to schedule the transmission of *sub-beacon* (s-beacon) frames to avoid collisions between multiple s-beacon transmissions.

An s-beacon shall provide synchronization locally and carry the ID of the parent AP, the ID of the transmitter and a bit field denoting whether the device is an AP or not (to facilitate the joining procedure of newcomer and legacy devices). For a sensor coordinating a sub-network such a bit shall be set to zero. In sensor networks transmitting sporadic data (in the order of minutes or even hours), it is reasonable to assume long periods of inactivity. Thus, each sub-network should take place during the inactive period within the superframe of the AP. When sensors located inside the cluttered region start receiving s-beacons, they have to select the sub-networks they receive with the strongest power, carry out association and reserve resources during the CFP using CSMA-CA within the CAP period. After collecting data from the associated cluttered sensors, each beaconing sensor will do the relay to the AP using the reserved GTS.

III. System Model

We study an infinite network of sensors with the AP centered at the origin the reference system as showed in Fig. 1. Furthermore, we assume the channel between a sensor and the AP affected by the general Nakagami distributed fading. We focus in the 2-dimensional plane on the area of a disk of radius R around the AP. Since the AP is the recipient of the whole network traffic, it is also in charge of monitoring whether all sensors report data or not. In case of an obstruction, we identify a region denoted by $\mathcal{A}_l$ in which a group of sensors spatially collocated cannot communicate with the AP. We notice that the cluttered region can have any shape (possibly irregular) and study connectivity can be hard in general. Thus, we assume $\mathcal{A}_l$ to be a convex region modeled as a regular polygon with l sides to take into account shape irregularity. Let $\mathcal{A}_r$ be a disk of radius $r \leq R$ inscribing the polygon and the outer region the set of points $\mathcal{A}_o := \mathcal{A}_r \setminus \mathcal{A}_l$.

In this work, we consider that sensors are distributed over space according to a homogeneous PPP. This choice is motivated by the fact that we do not target any specific sensor installation and therefore a PPP provides a generalization that allows also mathematical tractability. To study the tradeoff, we rely on the Slyvniak's theorem [9] that allows to exclude the point where the AP is located from the PPP. For a bounded Borel set B, the probability to find k nodes is

$$P_k = \frac{(\lambda_s \nu_d(B))^k}{k!} \exp\left(-\lambda_s \nu_d(B)\right), \tag{1}$$

where $\nu_d(B)$ is the d-dimensional Lebesgue measure of set B and λ_s is the constant intensity of the PPP. For a PPP Φ over the set B, the average number of points is $\mathbb{E}\Phi(B) = \lambda_s \nu_d(B)) = N$. The volume of a d-dimensional ball is $\nu_d(B)) = \frac{\pi^{d/2}}{\Gamma(1+d/2)} R^d$ and $\Gamma(\cdot)$ is the Gamma function. In the remainder, for the d-dimensional ball of radius R we assume a unit volume $\nu_d(B) = 1$ and hence that $\lambda_s = N$.

Lemma 1 (Connectivity in relay network): When on average $N + M$ nodes (N sensors and M relays) with transmission radius r are deployed over a disk of radius R according to a homogeneous PPP of intensity $\lambda_s = \lambda_{ss} + \lambda_{sr}$, which is the superposition of two independent homogeneous PPPs with individual intensities λ_{ss} and λ_{sr} respectively, the number of relays to enable network connectivity with is

$$M \leq -N - \left(\frac{R}{r}\right)^2 W_L\left(\left(\frac{r}{R}\right)^2 \log(1-\epsilon)\right), \tag{2}$$

with the equal sign that holds for a fixed target connectivity threshold $1-\epsilon \in (0,1)$ and where W_L is the Lambert function. Notice that the Lambert function computed for $(r/R)^2 \log(1-\epsilon)$ returns negative values and in turn M is a positive number. The value of M to enable connectivity should never be less than the result provided in Theorem 3 of [4] for the transmission radius of a node within the unit-area disk. The proof of the lemma is postponed to the appendix.

Definition 1: When connectivity in an STP-WSN is recovered using NACRP, we define topology penalty the topological change of the network from pure star to mesh. The topology penalty is quantified by means of the factor $\eta < 1$.

Definition 2: Under the hypothesis that sensors can transmit s-beacon signals at a power $P_t \leq P_{max}$, we denote with $\theta \leq 1$ the fraction of the maximum transmitted power used to carry on the additional duty of transmitting s-beacon frames.

Remark: We aim to study whether a tradeoff exists between the fraction of the maximum transmitted power which is necessary to cover the cluttered region with s-beacons (energy penalty due to additional transmissions) and the topological change introduced by the NACRP.

IV. System Analysis

The IEEE 802.15.4k standard allows the sensors to transmit data directly to the AP using the GTS in every superframe. When some sensors lose connectivity and the NACRP is used, the drawback is that the topology morphs to resemble locally a mesh network. We denote with n_o the number of sensors selected by the AP for broadcasting s-beacons inside the region

$\mathcal{A}_o$. Let r_0 be the radius of the area covered with an s-beacon and β the power threshold related to the receiver sensitivity. To recover connectivity inside the cluttered region with s-beacon signals, we use the union of the regions $\mathcal{A}_i,\ i = 0, \cdots, n_o$,

$$\mathcal{A}_l \subseteq \bigcup_{i=0}^{n_o} \mathcal{A}_i. \tag{3}$$

To develop subsequent calculations, we remind that the Lebesgue measure (i.e. the area) of a regular polygon with l sides is $|\mathcal{A}_l| = (l/2)\, r^2 \sin(2\pi/l)$, where $r = b_l/(2\sin(\pi/l))$ is the circum-radius, $|\mathcal{A}_i|$ is the area covered with the ith s-beacon signal and b_l is the polygon side length.

We assume an s-beacon signal corrupted by Nakagami distributed fading with severity index m and the probability to receive an s-beacon is $\mathbb{P}\left\{|h|^2 P_t r_0^{-\alpha} \geq \beta\right\}$, where $|h|^2$ is the power gain coefficient which accounts for the propagation through the radio channel and α is the power loss exponent. In other words, a beacon signal is correctly detected with a probability of detection $P_d(r_0)$ that can be written as follows

$$P_d(r_0) = 1 - \frac{\gamma(m, \beta r_0^{\alpha}/(\theta P_{\max}))}{\Gamma(m)}, \tag{4}$$

where $\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$ is the lower incomplete Gamma function. For any target value of P_d^* that our system needs to achieve, equation (4) can be inverted numerically.

Theorem 1: In an STP-WSN where sensors are scattered according to a homogeneous PPP of intensity λ_s and the NACRP is used to recover connectivity in a region with cluttered sensors modeled with a regular polygon of l sides, the transmitted power of the s-beacon has to be scaled by a factor θ^*, such as

$$\theta^*(p, l, N) = \tag{5}$$

$$\frac{\beta}{P_{\max}} \frac{1}{\gamma_{\text{inv}}\left((1 - P_d^*)\Gamma(m), m\right)} \frac{\left(\frac{b_l^2 \text{sinc}(2/l)}{4\sin^2(\pi/l)}\right)^{\alpha/2}}{\left(1 + pN\pi b_l^2 \frac{1-\text{sinc}(2/l)}{4\sin^2(\pi/l)}\right)^{\alpha/2}},$$

with p the probability a sensor is selected to transmit s-beacons, sinc(x):=sin$(\pi x)/\pi x$ is the *sinc* function and γ_{inv} is the inverse of the lower incomplete Gamma function.

Theorem 2: Relying on the same general hypotheses provided in Theorem 1, the topology penalty due to the use of s-beacons is

$$\eta(p, l, N) = p\left(1 - \frac{1 - \exp\left(-\pi N b_l^2 \frac{1-\text{sinc}(2/l)}{4\sin^2(\pi/l)}\right)}{N\left(b_l^2 \frac{1-\text{sinc}(2/l)}{4\sin^2(\pi/l)}\right)}\right). \tag{6}$$

The proof of both theorems is postponed to the appendix.

V. Results

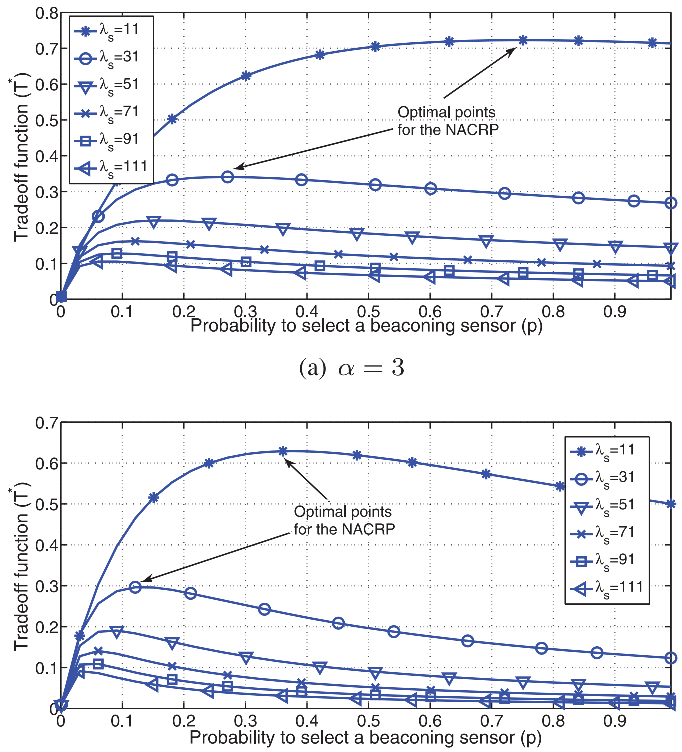
To explain the tradeoff between the number of sensors selected to transmit s-beacons and the scaling of the transmitted power, we introduce the following normalized function

$$\mathcal{T}^*(p, l, N) := \kappa^{-1} \times \theta^*(p, l, N) \times \eta(p, l, N), \tag{7}$$

where $\kappa^{-1} = \max_p \left\{\mathcal{T}^*(p, \hat{l}, \hat{N})\right\}$ is the normalization constant for fixed $\hat{l}$, $\hat{N}$ values. The normalized function $0 \leq \mathcal{T}^* \leq 1$ denotes the desired set of tradeoff points arising from the number of sub-networks to cover the cluttered region and the transmission power of s-beacons as a function of p, l and N. To obtain the inversion of the lower incomplete Gamma function we use the following polynomial expansion: $1 - \frac{\gamma(a,x)}{\Gamma(a)} = P_d^*$, where $\gamma(a, x) = x^a \sum_{s=0}^{\infty} \frac{(-x)^s}{s!(a+s)}$. To show results, we have truncated the infinite summation with a polynomial degree $n_{\text{Poly}} = 10 + m$ to match $P_d^* = 0.9$. The only valid solution is obtained for $x = x^* \in \mathbb{R}^+$. Furthermore, the side length b_l is set to fulfill the condition $r \leq 1/\sqrt{\pi}$ in the unit area disk. As in a realistic setting $\beta/P_{\max} << 1$.

TABLE I
SYSTEM PARAMETERS

Parameter	Comment	Value
P_d^*	Detection probability	0.9
N	Number of sensors	200
α	Power loss exponent	{3, 4}
$\beta/P_{\max}$	Condition for detection	0.008
l	polygon region shape	3 ÷ 20 sides



(a) $\alpha = 3$

(b) $\alpha = 4$

Fig. 2. Tradeoff function $\mathcal{T}^*$ vs. probability p for $l = 3$ sides.

Numerical values we use are provided in Table I. Figure 2(a) and Figure 2(b) show the tradeoff function $\mathcal{T}^*$ respectively for $\alpha = 3$ and $\alpha = 4$, as a function of the probability p and different values of the intensity λ_s selecting $l = 3$ sides for the cluttered region. Both figures show the existence of an optimal tradeoff point for the function $\mathcal{T}^*$. The existence of an optimal point (maximum value of $\mathcal{T}^*$) reflects that, fixing λ_s, equation (5) decreases as p increases, while equation (6) grows linearly with p. Interestingly, increasing the density of the nodes λ_s the network becomes denser and the fraction of sensors to use in the NACRP can be reduced. Results show the existence of a probability $p = p^*$ in correspondence of the optimal point. Anyway, to spare sensors' battery, it could be preferable to use a value of p even greater than p^* (higher topological penalty due to the use of more sensors) but transmitting s-beacons with a power which can be further lowered properly selecting θ^*. Finally, the optimal value p^* can be exploited to compute the density of relays $\lambda_{\rm sr} = \lambda_{\rm s} p^*$ to be used in Lemma 1.

VI. Conclusions

In this work, we have investigated an STP-WSN compliant with the recent IEEE 802.15.4k standard amendment. We have presented the NACRP, a novel protocol to recover from connectivity loss when the direct link between one or more sensors and the AP is cluttered by the sudden appearance of temporary obstructions. We have analyzed the protocol resorting to the tool of stochastic geometry to characterize the spatial process of sensors scattered over a survey area. Whereby our analysis we managed to identify the set of tradeoff points between the power that sensors have to spend to transmit s-beacon frames and the topological change due to the fact that the star topology morphs into local mesh networks. In particular, we have showed that optimal tradeoff points can be identified for different propagation characteristics.

Appendix
Proof of Lemma 1

We focus on a reference sensor inside a disk of radius $r \leq R$. Relying on the Slyvniak's theorem, such a point is not counted in the subsequent calculations. Using the stationary property of PPPs, we can write: $M_r = \lambda_{\rm sr}|\mathcal{A}_r| = qM$ and $N_r = \lambda_{\rm ss}|\mathcal{A}_r| = qN$, with $q = (r/R)^2$. For a PPP, the probability that the reference sensor is isolated can be written as: $\exp\left(-(N_r + M_r)\right)$. The probability the reference sensor is not isolated is the complementary probability. Conditioning on the number k of points of the PPP and then removing the conditioning we can write

$$\sum_{k=0}^{\infty} (1 - \exp\left(-(N_{\rm r} + M_{\rm r})\right))^k \frac{(\lambda_{\rm s}|\mathcal{A}_{\rm R}|)^k}{k!} \exp\left(-\lambda_{\rm s}|\mathcal{A}_{\rm R}|\right)$$
$$= \exp\left(-(N + M)e^{-q(N+M)}\right). \tag{8}$$

The threshold for connectivity is $1 - \epsilon$, taking the logarithm and multiplying by q we obtain

$$-q(N + M)e^{-q(N+M)} \geq q \log\left(1 - \epsilon\right). \tag{9}$$

Equation (9) is of type $\Delta(z)e^{\Delta(z)}$, which is the Lambert equation with $\Delta(z) = -q(N + M)$. The Lemma can be proved from $-q(N + M) \geq W_L\left(q \log(1 - \epsilon)\right)$, with W_L the Lambert function.

Proof of Theorem 1

We rely here on the condition established in (3) to say that

$$(1 + n_{\rm o})\pi r_0^2 \geq |\mathcal{A}_l| , \tag{10}$$

where r_0 is the radius of the area covered with an s-beacon and at least one node is available in the region $|\mathcal{A}_{\rm o}|$. We condition first on the number of sensors $n_{\rm o}$ selected by the AP according to the NACRP in the outer region $\mathcal{A}_{\rm o}$. This is a Binomial random variable (r.v.) with probability p. Conditioning also on the number of nodes n inside $\mathcal{A}_{\rm o}$, we are able to compute the expected number of sensors as $1 + pn$, with n a Poisson distributed r.v. Relying on Section IV, the area $|\mathcal{A}_{\rm o}|$ of the region is computed as follows

$$|\mathcal{A}_{\rm o}| = |\mathcal{A}_r| - |\mathcal{A}_l| = \pi r^2 \left(1 - \frac{l}{2\pi} \sin\left(\pi \frac{2}{l}\right)\right)$$
$$r = b_l / \left(2\sin(\pi/l)\right) . \tag{11}$$

Rearranging the terms in (11), we obtain $|\mathcal{A}_{\rm o}| = \pi b_l^2 \left(\frac{1-{\rm sinc}(2/l)}{4\sin^2(\pi/l)}\right)$. To remove also the conditioning on the Poisson distributed r.v. n, we exploit that the number of nodes in non-overlapping regions are independent and are only a function of the d-dimensional volume of each region. We thus replace $\nu_d(B)$ with $|\mathcal{A}_{\rm o}|$ in equation (1) to obtain $(1 + p\lambda_{\rm s}|\mathcal{A}_{\rm o}|)$, where $\lambda_{\rm s} = N$. Therefore, using (11) we obtain

$$r_0 = \left(\frac{1}{1 + pN|\mathcal{A}_{\rm o}|}\right)^{1/2} \left(b_l^2 \frac{{\rm sinc}(2/l)}{4\sin^2(\pi/l}\right)^{1/2} . \tag{12}$$

The proof is completed inverting equation (4) with respect to r_0 in correspondence of $P_{\rm d} = P_{\rm d}^*$.

Proof of Theorem 2

Since n_o is the r.v. of the number of nodes within the outer region $\mathcal{A}_{\rm o}$, assumed Binomial with probability p, conditioning on the number of nodes n, on average pn sensors can be found in the outer region. Sensors can be selected with the same equal probability out of $n + 1$ and since n is a Poisson distributed r.v. we can write

$$\sum_{n=0}^{\infty} pn \frac{1}{n+1} \frac{(\lambda_{\rm s}|\mathcal{A}_{\rm o}|)^n}{n!} \exp(-\lambda_{\rm s}|\mathcal{A}_{\rm o}|) = \tag{13}$$
$$= \frac{p}{\lambda_{\rm s}|\mathcal{A}_{\rm o}|} e^{-\lambda_{\rm s}|\mathcal{A}_{\rm o}|} \sum_{j=1}^{\infty} (j-1) \frac{(\lambda_{\rm s}|\mathcal{A}_{\rm o}|)^j}{j!} ,$$

where we did the change of index $n + 1 = j$. Considering that $\lambda_{\rm s} = N$, after simple manipulations the proof is completed.

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