Wireless Sensor Network Localization in Harsh Environments Using SDP Relaxation

Pouya Mollaebrahim Ghari, Reza Shahbazian, *Student Member, IEEE*, and Seyed Ali Ghorashi, *Senior Member, IEEE*

***Abstract*—The accuracy of localization in wireless sensor networks depends on noise level and the presence of nonline of sight (NLOS) connections. In this letter, we propose a novel semidefinite programming (SDP) method to improve the accuracy and to reduce the required time for wireless sensor network localization in harsh environments. In fact, we perturb Edge-based SDP (ESDP) relaxation and add some constraints to the optimization problem in order to make the proposed localization method robust against large amounts of error in distance measurements. Simulation results confirm that our proposed method outperforms others when the majority of connections are NLOS, noise level is high and it is not possible to distinguish between NLOS and line of sight (LOS) connections.**

***Index Terms*—Localization, wireless sensor network, semidefinite programming.**

I. Introduction

NOWADAYS, Wireless Sensor Networks (WSNs) are considered to provide reliable solutions to a wide variety of applications. In general, by knowing the location of a node, more meaningful data may be collected by a WSN. Therefore, in some applications, localization is a necessity. Although the Global Positioning System (GPS) can be used to determine the position of sensors, this could be expensive or even impossible in some cases [1]. In such cases, the location of each node can be estimated based on the distance measurements of neighboring nodes. In addition, there are a few nodes with known positions (called anchors) that can be used to solve the localization problem.

Localization problem can be described as follows. Here, we consider a two-dimensional (2D) case whose extension to higher dimensions is straightforward. There are n sensors with unknown locations $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$ and m anchors whose locations are denoted by $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m$. We also denote the Euclidean distance between a pair of sensors $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ by $d_{\mathrm{s},ij}$, and it is defined only when the distance between them is less than a specific radio range. Similarly, for a sensor $\boldsymbol{x}_j$ and an anchor $\boldsymbol{a}_k$, the Euclidean distance is denoted by $d_{\mathrm{a},jk}$. Then, the localization problem in a WSN can be expressed as follows:

$$\text{Find } \boldsymbol{X} \in \mathcal{R}^{2\times n} \tag{1a}$$
$$\text{s.t. } \boldsymbol{Y}_{ii} - 2\boldsymbol{Y}_{ij} + \boldsymbol{Y}_{jj} = d_{\mathrm{s},ij}^2, \quad \forall (j,i) \in N_{\mathrm{s}} \tag{1b}$$
$$\boldsymbol{Y}_{jj} - 2\boldsymbol{x}_j^T \boldsymbol{a}_k + \|\boldsymbol{a}_k\|^2 = d_{\mathrm{a},jk}^2, \quad \forall (j,k) \in N_{\mathrm{a}} \tag{1c}$$
$$\boldsymbol{Y} = \boldsymbol{X}^T \boldsymbol{X} \tag{1d}$$

where $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n]$, $N_{\mathrm{s}} = \left\{(j,i) \mid \left\|\boldsymbol{x}_j - \boldsymbol{x}_i\right\| < r\right\}$, $N_{\mathrm{a}} = \left\{(j,k) \mid \left\|\boldsymbol{x}_j - \boldsymbol{a}_k\right\| < r\right\}$ and r is the radio range. Convex relaxation techniques provide powerful approaches to solve the sensor network localization problem. Semi-Definite Programming (SDP) relaxation that has been proposed in [2] can transform the non-convex problem (1) into a convex one. Authors in [3] modified SDP in order to find a low rank solution, but this method could not provide more accuracy compared with full-SDP introduced in [2]. In such approaches, the constraint (1d) is relaxed to the following Linear Matrix Inequality (LMI):

$$\boldsymbol{Y} \succeq \boldsymbol{X}^T\boldsymbol{X} \rightarrow \boldsymbol{Z} = \begin{pmatrix} \mathbf{I}_2 & \boldsymbol{X}^T \\ \boldsymbol{X} & \boldsymbol{Y} \end{pmatrix} \succeq 0 \tag{2}$$

where $\mathbf{I}_n$ denotes n by n identity matrix. Edge-based Semi-Definite Programming (ESDP) relaxation has less computational complexity than the full-SDP and at the same time it is comparable with full-SDP in terms of accuracy [4]. By applying ESDP relaxation to the problem (1), the localization problem can be written as follows:

$$\min_{\substack{\boldsymbol{\alpha}^+,\boldsymbol{\alpha}^-,\boldsymbol{\beta}^+,\\ \boldsymbol{\beta}^-,\boldsymbol{Z},\boldsymbol{Y}}} \sum_{(j,i)\in N_{\mathrm{s}}} \left(\alpha_{ij}^+ + \alpha_{ij}^-\right) + \sum_{(j,k)\in N_{\mathrm{a}}} \left(\beta_{jk}^+ + \beta_{jk}^-\right) \tag{3a}$$
$$\text{s.t.} \quad \boldsymbol{Z}_{(1,2),(1,2)} = \mathbf{I}_2 \tag{3b}$$
$$\boldsymbol{Y}_{ii} - 2\boldsymbol{Y}_{ij} + \boldsymbol{Y}_{jj} - \alpha_{ij}^+ + \alpha_{ij}^- = d_{\mathrm{s},ij}^2 \tag{3c}$$
$$\boldsymbol{Y}_{jj} - 2\boldsymbol{x}_j^T\boldsymbol{a}_k + \|\boldsymbol{a}_k\|^2 - \beta_{jk}^+ + \beta_{jk}^- = d_{\mathrm{a},jk}^2$$
$$\forall (j,i) \in N_{\mathrm{s}}, \quad \forall (j,k) \in N_{\mathrm{a}} \tag{3d}$$
$$\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)} \succeq 0, \quad \forall (j,i) \in N_{\mathrm{s}} \tag{3e}$$
$$\boldsymbol{\alpha}^+, \boldsymbol{\alpha}^-, \boldsymbol{\beta}^+, \boldsymbol{\beta}^- \geq 0 \tag{3f}$$

where $\boldsymbol{\alpha}^+, \boldsymbol{\alpha}^-, \boldsymbol{\beta}^+$ and $\boldsymbol{\beta}^-$ are squared distance errors. $\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)}$ is a sub-matrix of $\boldsymbol{Z}$ consists of rows and columns $1, 2, i, j$. In [5], Edge-based Maximum Likelihood (EML) relaxation has been proposed in order to enhance the performance of ESDP relaxation. All mentioned methods in [2]–[5] have been presented for the case that all pairs of nodes have Line of Sight (LOS) links. However, some connections particularly in indoor networks are Non Line of Sight (NLOS).

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P. M. Ghari and R. Shahbazian are with the Department of Electrical Engineering, Shahid Beheshti University, Tehran 1983969411, Iran (e-mail: p.molaebrahim@mail.sbu.ac.ir; r_shahbazian@sbu.ac.ir).

S. A. Ghorashi is with the Department of Electrical Engineering and Cyberspace Research Institute, Shahid Beheshti University, Tehran 1983969411, Iran (e-mail: a_ghorashi@sbu.ac.ir).

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SDP-based methods have been employed to localize sensors in indoor networks [6], [7]. An SDP-based method, namely SDP-M has been proposed in [6] to show that NLOS connections can be exploited to enhance the accuracy of the localization. They assume that NLOS connections are identifiable while this may not be a practical assumption in some cases [7]. Authors of [7] have proposed a localization method for cases that there is no information about NLOS connections and also they are not distinguishable from LOS connections.

Errors in distance measurements are usually modeled by the following formulation [7]:

$$\hat{d}_{ij} = d_{ij} + G(0, \sigma_{ij}^2) + b_{ij}, \quad \forall (j,i) \in N_s \cup N_a \cup NL_s \cup NL_a \tag{4}$$

where $\{b_{ij}\}$ is the set of unknown positive NLOS biases and $\{d_{ij}\}$ includes both $\{d_{s,ij}\}$ and $\{d_{a,jk}\}$. $G(0, \sigma_{ij}^2)$ denotes a Gaussian distribution with zero mean and variance of $\sigma_{ij}^2 = K_E d_{ij}^{\gamma}$. NL_s and NL_a include NLOS sensor-sensor and sensor-anchor connections, respectively. Note that K_E is the scaling parameter that determines the accuracy of distance measurements. This model for accuracy of distance measurements can be applied to both Time of Arrival (TOA) and Received Signal Strength (RSS) based measurements when $\gamma = 2$ as we consider in this letter.

In this letter, we modify ESDP in order to find a low rank solution and introduce a new class of SDP-based relaxation that is robust against high level noise and NLOS biases. We evaluate our method based on the assumption that the estimator does not know which connections are NLOS. Furthermore, we consider the case in which the statistics of $\{b_{ij}\}$ are unknown.

II. Proposed Convex Relaxation

In this letter, we aim to propose an SDP relaxation to enhance the performance of the SDP-based localization methods in harsh environments. When there is no corruption in measured distances, we have:

$$(\boldsymbol{e_i} - \boldsymbol{e_j}; \boldsymbol{0})^T \boldsymbol{Z}^{(\text{true})} (\boldsymbol{e_i} - \boldsymbol{e_j}; \boldsymbol{0}) = d_{s,ij}^2, \forall (i,j) \in N_s \tag{5a}$$

$$(\boldsymbol{e_j}; -\boldsymbol{a_k})^T \boldsymbol{Z}^{(\text{true})} (\boldsymbol{e_j}; -\boldsymbol{a_k}) = d_{a,jk}^2, \forall (j,k) \in N_a \tag{5b}$$

where $\boldsymbol{Z}^{(\text{true})}$ is the $\boldsymbol{Z}$ matrix in the absence of any corruption and $\boldsymbol{e_i}$ is a zero column vector except for a one in the position associated with index i. In the presence of measurement noise, the constraints (3c) and (3d) can be rewritten as follows:

$$(\boldsymbol{e_i} - \boldsymbol{e_j}; \boldsymbol{0})^T \boldsymbol{Z}^{(\text{n})} (\boldsymbol{e_i} - \boldsymbol{e_j}; \boldsymbol{0}) + \alpha_{ij}^+ - \alpha_{ij}^- = d_{s,ij}^2 + t_{ij} \tag{6a}$$

$$(\boldsymbol{e_j}; -\boldsymbol{a_k})^T \boldsymbol{Z}^{(\text{n})} (\boldsymbol{e_j}; -\boldsymbol{a_k}) + \beta_{jk}^+ - \beta_{jk}^- = d_{a,jk}^2 + v_{jk} \tag{6b}$$

$$t_{ij} = 2n_{ij} d_{s,ij} + n_{ij}^2, \quad \forall (i,j) \in N_s \tag{6c}$$

$$v_{jk} = 2\delta_{jk} d_{a,jk} + \delta_{jk}^2 \quad \forall (j,k) \in N_a \tag{6d}$$

where the noise terms associated with $d_{s,ij}$ and $d_{a,jk}$ are denoted by n_{ij} and δ_{jk}, respectively, and $\boldsymbol{Z}^{(\text{n})}$ denotes the noisy $\boldsymbol{Z}$ matrix. Errors caused by NLOS connections are positive and generally greater than the measurement noise [6]. Therefore, considering NLOS distance measurements as upper bound in the optimization problem can be helpful. Furthermore, when each pair of nodes has a LOS link, based on Chebyshev's inequality, more than 75% of distance measurements are not greater than $\text{mean}(\{\hat{d}_{ij}\}) + 2 \times \text{std}(\{\hat{d}_{ij}\})$. In NLOS environments, due to the presence of large positive biases, both $\text{mean}(\{\hat{d}_{ij}\})$ and $\text{std}(\{\hat{d}_{ij}\})$ become considerably greater than the case that all connections are LOS. Therefore, in this case the probability that a LOS distance measurement be greater than $\text{mean}(\{\hat{d}_{ij}\}) + 2 \times \text{std}(\{\hat{d}_{ij}\})$ is low. According to (6), considering LOS distance measurements as upper bound restricts the feasibility set, and this may increase the optimal matrix. As it will be shown in this section, by using our proposed relaxation, feasibility set is expanded and as a result the optimal value may reduce. Because of this, we prefer to consider wider proportion of distances as upper bounds and add the following constraints to the optimization problem when NLOS connections are not identifiable:

$$(\boldsymbol{e_i} - \boldsymbol{e_j}; \boldsymbol{0})^T \boldsymbol{Z}^{(\text{n})} (\boldsymbol{e_i} - \boldsymbol{e_j}; \boldsymbol{0}) \le \hat{d}_{s,ij}^2, \tag{7a}$$

$$(\boldsymbol{e_j}; -\boldsymbol{a_k})^T \boldsymbol{Z}^{(\text{n})} (\boldsymbol{e_j}; -\boldsymbol{a_k}) \le \hat{d}_{a,jk}^2, \tag{7b}$$

$$\{\hat{d}_{s,ij}, \hat{d}_{a,jk}\} \ge \text{mean}(\{\hat{d}_{ij}\}) + \text{std}(\{\hat{d}_{ij}\})$$

From (5), (6) and (7), we can conclude that errors in distance measurements can cause perturbation in $\boldsymbol{Z}$ and this degrades the accuracy of localization. Therefore, $\boldsymbol{Z}$ can be expressed as follows:

$$\boldsymbol{Z}^{(\text{n})} = \boldsymbol{Z}^{(\text{true})} + \boldsymbol{\Delta} \tag{8}$$

When the constraints (7a) and (7b) are added to the problem (3), the dual of the resulted problem can be written as follows:

$$\max_{\boldsymbol{S},\boldsymbol{u},\boldsymbol{\omega}} \quad \Big(g(\boldsymbol{u},\boldsymbol{\omega}) = u_{11} + 2u_{12} + u_{22} + \sum_{(i,j)\in N_s \cup NL_s} \omega_{s,ij}(d_{s,ij} + n_{ij} + b_{ij})^2 + \sum_{(j,k)\in N_a \cup NL_a} \omega_{a,jk}(d_{a,jk} + \delta_{jk} + b_{jk})^2\Big) \tag{9a}$$

$$\text{s.t.} \quad \sum_{(i,j)\in N_s \cup NL_s} \{\omega_{s,ij}(\boldsymbol{0}; \boldsymbol{e_i} - \boldsymbol{e_j})^T(\boldsymbol{0}; \boldsymbol{e_i} - \boldsymbol{e_j}) + \boldsymbol{S}^{(i,j)}\} + \sum_{(j,k)\in N_a \cup NL_a} \omega_{a,jk}(-\boldsymbol{a_k}; \boldsymbol{e_j})^T(-\boldsymbol{a_k}; \boldsymbol{e_j}) + \begin{pmatrix} u_{11}+u_{12} & u_{12} & \boldsymbol{0} \\ u_{12} & u_{22}+u_{12} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix} = 0 \tag{9b}$$

$$\omega_{s,ij} \le 0, \forall (j,i) \in NL_s, \quad \omega_{a,jk} \le 0, \forall (j,k) \in NL_a \tag{9c}$$

$$\boldsymbol{S}_{(1,2,i,j),(1,2,i,j)}^{(i,j)} \succeq 0, \quad \forall (j,i) \in N_s \cup NL_s \tag{9d}$$

$$\boldsymbol{S}_{kl}^{(i,j)} = 0, \quad \forall k \notin \{i,j\} \text{ or } l \notin \{i,j\} \tag{9e}$$

where $\boldsymbol{S}$, $\boldsymbol{u}$ and $\boldsymbol{\omega}$ are the dual variables of the problem (3). In this section, NL_s and NL_a include pairs of nodes that satisfy (7). By perturbation and sensitivity analysis, we may write [8]:

$$\nabla_{\boldsymbol{\Delta}_{(1,2,i,j),(1,2,i,j)}} f^*(0,0,0) = \boldsymbol{S}_{(1,2,i,j),(1,2,i,j)}^{(i,j)^*} \tag{10}$$

The optimal value associated with (6) and (7) is $f^*(\boldsymbol{t}, v, \boldsymbol{\Delta})$ and $\boldsymbol{S}_{(1,2,i,j),(1,2,i,j)}^{(i,j)^*}$ denotes the optimal value of the problem

(9). This means that if the absolute values of elements in $\boldsymbol{S}$ decrease effectively, $f^*(\boldsymbol{t}, v, \boldsymbol{\Delta})$ does not increase rapidly in the presence of error. To do this, we modify ESDP relaxation by a perturbation matrix $\boldsymbol{P}$ as follows:

$$\min_{\substack{\boldsymbol{\alpha}^+,\boldsymbol{\alpha}^-,\boldsymbol{\beta}^+,\\ \boldsymbol{\beta}^-,\boldsymbol{Z},\boldsymbol{Y}}} \quad (3a) \tag{11a}$$

$$\text{s.t.} \quad (3b), (3c), (3d), (3f), (7a), (7b) \tag{11b}$$

$$\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)} + \boldsymbol{P}_{(1,2,i,j),(1,2,i,j)} \succeq 0 \quad \forall(i, j) \in N_{\text{s}} \cup NL_{\text{s}} \tag{11c}$$

Constraints in dual problem of (11) are the same as constraints in (9). However, the objective function is as follows:

$$g(\boldsymbol{u}, \boldsymbol{\omega}) - \sum_{(i,j)\in N_{\text{s}}\cup NL_{\text{s}}} \text{tr}\left(\boldsymbol{P}_{(1,2,i,j),(1,2i,j)}\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)}\right) \tag{12}$$

where $g(\boldsymbol{u}, \boldsymbol{\omega})$ is defined in (9a). Now, the optimal value of the problem (3) is made robust against perturbation in $\boldsymbol{Z}^{(\text{n})}$ by using the relaxation in (11c). It is useful to note that in prior SDP problems, eigenvalues of $\boldsymbol{Z}$ are positive while eigenvalues of $\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)}$ in the problem (11) can be negative. This expands the feasibility set of the localization problem in order to obtain a lower optimal value.

Required solution time for WSN localization problem depends on the rank of $\boldsymbol{Z}$. Finding a low rank solution may reduce the solution time. Therefore, we determine the perturbation matrix $\boldsymbol{P}$ in order to find a low rank solution. Assume that $\boldsymbol{Z}$ and $\{\boldsymbol{S}^{(i,j)}\}$ are solutions of the problem (11) and its dual, respectively. Then we have:

$$\text{rank}(\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)}) + 4 \leq 2q \Rightarrow \text{rank}(\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)}) \leq q \tag{13}$$

The proof of (13) is given in Appendix A. From (13), we can conclude that by minimizing the rank of $\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)}$, we minimize the rank of $\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)}$ as well.

Since we aim to find a low rank solution, we use rank(.) as a regularizer. It is known that if a matrix is symmetric and positive semidefinite, we may minimize its trace as a convex approximation of its rank. Therefore, we aim to determine the perturbation matrix $\boldsymbol{P}_{(1,2,i,j),(1,2,i,j)}$ in order to minimize the rank of $\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)}$. Thus, the perturbation matrix is chosen as follows:

$$\boldsymbol{P}_{(1,2,i,j),(1,2,i,j)} = p_{ij}\mathbf{I}_4, \quad \forall(j, i) \in N_{\text{s}} \cup NL_{\text{s}} \tag{14}$$

Then, we may rewrite (12) as follows:

$$g(\boldsymbol{u}, \boldsymbol{\omega}) - \sum_{(i,j)\in N_{\text{s}}\cup NL_{\text{s}}} p_{ij} \ \text{tr}(\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)}) \tag{15}$$

Therefore, by the perturbation matrix in (14), we can minimize the rank of $\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)}$. This minimizes the rank of $\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)}$.

Obviously, choosing an appropriate regularization term is an important issue in our proposed Perturbed Edge base Semi-Definite Programming (PESDP) method. If it is too low, the perturbation matrix will not affect the ESDP solution, effectively. On the other hand, if it is chosen too large, $\text{tr}(\boldsymbol{Y} - \boldsymbol{X}^T\boldsymbol{X})$ will become so large that accuracy of the localization may degrade, significantly. The optimum choice of the parameter p_{ij} depends on the size of the network and the level of noise. Our numerical results show that the optimum choice of regularization term varies with errors in distance measurements. It is also shown that the number of sensors does not affect the localization accuracy, considerably, in the presence of large errors in distance measurements.

III. Simulation Results

In this section, several numerical comparisons for our proposed formulation in (11) are reported. The aim of this section is to show how perturbed SDP relaxation performs under various error conditions. The simulations are performed in a 20 m by 20 m area. In addition, we use SDPT3 solver in CVX software [9]. We compute the average position error over 50 realizations of noise and biases as follows:

$$PE = (1/nL)\left(\sum_{j=1}^{L}\sum_{i=1}^{n}\|\hat{\boldsymbol{x}}_i - \boldsymbol{x}_i\|\right) \tag{16}$$

where L is the number of noise realizations and n is the number of sensors. $\hat{\boldsymbol{x}}_i$ and $\boldsymbol{x}_i$ denote the estimated and true positions of the ith sensor, respectively. The maximum number of neighbors for each sensor is limited to 5. $\{b_{ij}\}$ is exponentially distributed and NLOS connections are not identifiable. The proposed method in [7] is called SDP-NLOS in this letter, for comparison. For LOS connections, we have $E[\hat{d}^2_{ij}] = E[d^2_{ij}] + E[\xi^2_{ij}]$ for all $(j, i) \in N_{\text{s}} \cup N_{\text{a}}$ where $\boldsymbol{\xi}$ is the zero mean error vector. Therefore, we can conclude that the greater variance of additive noise, the greater mean of squared distance measurements. The value of p_{ij} should be increased by the growth in noise level and NLOS biases. Therefore, we set the value of p_{ij} proportional to $E[\hat{d}^2_{ij}]$. By our numerical experiences, we realize that $0.5r^2$ can be used as a criterion to determine how large $E[\hat{d}^2_{ij}]$ is. When $E[\hat{d}^2_{ij}]$ is less than $0.5r^2$, we set p_{ij} to 0. Otherwise, we obtain p_{ij} from the following formulation:

$$p_{ij} = 0.05(E[\hat{d}^2_{ij}/r^2] - 0.5) \quad \forall(i, j) \in N_{\text{s}} \cup N_{\text{a}} \cup NL_{\text{s}} \cup NL_{\text{a}} \tag{17}$$

In Fig. 1 the effect of NLOS connections on the position error and the standard deviation of error is studied. μ_N denotes the ratio of the number of NLOS connections to the number of all available distance measurements. K_E, the radio range and mean of NLOS biases are set to 0.1, 6 m and 5 m, respectively. The number of sensors and anchors are 50 and 5, respectively. As shown in Fig. 1 our proposed PESDP outperforms the other methods in terms of accuracy for all amounts of μ_N. Note that, in Fig. 1 "PESDP0" denotes the problem (11) with $p_{ij} = 0$. Fig. 1 displays the improvement caused by perturbed SDP relaxation (11c).

In Fig. 2 we evaluate the performance of localization methods by increasing the noise value, for n = 100. 40% of connections are in NLOS and the mean of NLOS biases is considered to be 4 m. The number of anchors is 5 and the radio

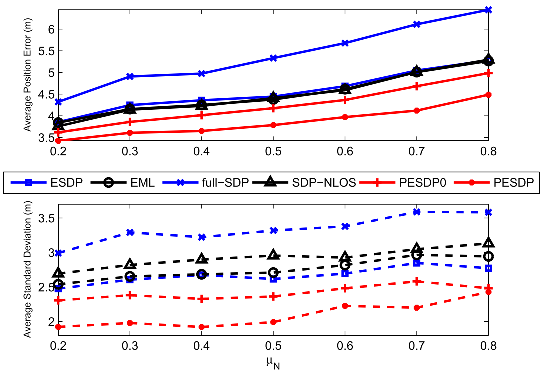

Fig. 1. Average position error (solid lines) and standard deviation (dashed lines) of ESDP [4], EML [5], full-SDP [2], SDP-NLOS [7] and our proposed PESDP when NLOS connections are not distinguishable.

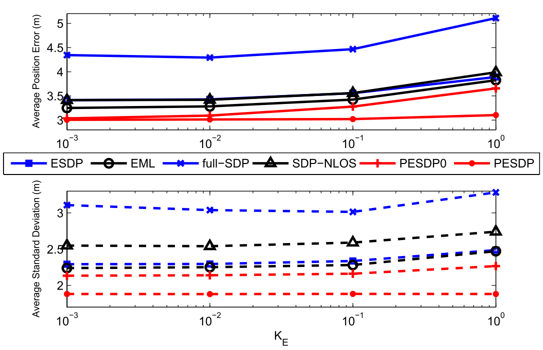

Fig. 2. Average position error (solid lines) and standard deviation (dashed lines) of ESDP [4], EML [5], full-SDP [2], SDP-NLOS [7] and our proposed PESDP by increasing the noise value when NLOS connections are not distinguishable.

TABLE I

ACCURACY COMPARISON IN A 40 m BY 40 m AREA

n	r	K_E	ESDP	FULL-SDP	EML	PESDP	$\sqrt{CRLB}$
200	6m	0.005	3.19m	3.18m	3.15m	3.19m	0.28m
400	4m	0.5	4.20m	4.36m	4.12m	3.88m	2.10m

range is set to 4 m. As can be seen in Fig. 2, our proposed PESDP shows better performance compared to the other methods. Simulations show that our proposed PESDP causes at least 0.3 m improvement in both average position error and average standard deviation compared to others. Table I presents the accuracy of localization methods. Simulations are performed in a 40 m by 40 m area and the number of anchors is set to 15. All connections are LOS. Furthermore, Cramer-Rao Lower Bound (CRLB) is computed using [1].

Table II shows the required time to solve the optimization problem of different edge-based algorithms for different number of sensors. For these results, the number of anchors is set to 5 and all nodes are considered to be in LOS. K_E and the radio range are set to 0.1 and 6 m, respectively. As can be seen in table II, our proposed PESDP is much faster than EML, especially when the number of sensors is large. It is also shown that our proposed PESDP is faster than ESDP.

IV. CONCLUSION

In this letter, we proposed a new SDP relaxation to enhance the performance of SDP-based localization methods in both accuracy and computational complexity. We modified EDSP relaxation by a perturbation matrix in order to make it robust against large errors in distance measurements. Simulation results confirmed that our proposed PESDP shows better accuracy compared to ESDP, EML, full-SDP and SDP-NLOS in NLOS environments. The performance of the proposed method is evaluated when NLOS and LOS connections are not identifiable.

TABLE II

SOLVING TIME COMPARISON IN SECONDS

n	ESDP	EML	PESDP
100	6.51s	9.56s	6.28s
200	16.13s	25.43s	14.84s
300	34.23s	52.42s	30.76s
400	81.72s	119.00s	73.59s

APPENDIX A

Assume that: $\boldsymbol{A} \prec 0$. Then, by (9d) we may write:

$$\boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)} - \boldsymbol{Z}^T_{(1,2,i,j),(1,2,i,j)}\boldsymbol{A}\boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)} \succeq 0 \tag{18}$$

Then, we may write the following using Schur complement:

$$\begin{pmatrix} \boldsymbol{S}^{(i,j)}_{(1,2,i,j),(1,2,i,j)} & \boldsymbol{Z}^T_{(1,2,i,j),(1,2,i,j)} \\ \boldsymbol{Z}_{(1,2,i,j),(1,2,i,j)} & \boldsymbol{A}^{-1} \end{pmatrix} \succeq 0 \tag{19}$$

Now consider the following lemma [10]: Let $\boldsymbol{U} \in \mathcal{R}^{m\times n}$ be a given matrix then $\text{rank}(\boldsymbol{U}) \le q$ if and only if there exist matrices $\boldsymbol{V} = \boldsymbol{V}^T \in \mathcal{R}^{m\times m}$ and $\boldsymbol{W} = \boldsymbol{W}^T \in \mathcal{R}^{n\times n}$ such that:

$$\text{rank}(\boldsymbol{V}) + \text{rank}(\boldsymbol{W}) \le 2q$$

$$\begin{pmatrix} \boldsymbol{V} & \boldsymbol{U} \\ \boldsymbol{U}^T & \boldsymbol{W} \end{pmatrix} \succeq 0 \tag{20}$$

Therefore, we may conclude (13).

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